Sains Malaysiana 41(11)(2012): 1403–1410

Preliminary Study on Bayesian Extreme Rainfall Analysis: A Case Study of Alor Setar, Kedah, Malaysia

(Kajian Awal bagi Analisis Kehujanan Melampau Bayes: Kajian Kes di Alor Setar, Kedah, Malaysia)

ANNAZIRIN ELI*, MARDHIYYAH SHAFFIE & WAN ZAWIAH WAN ZIN

ABSTRACT

Statistical modeling of extreme rainfall is essential since the results can often facilitate civil engineers and planners to estimate the ability of building structures to survive under the utmost extreme conditions. Data comprising of annual maximum series (AMS) of extreme rainfall in Alor Setar were fitted to Generalized Extreme Value (GEV) distribution using method of maximum likelihood (ML) and Bayesian Markov Chain Monte Carlo (MCMC) simulations. The weakness of ML method in handling small sample is hoped to be tackled by means of Bayesian MCMC simulations in this study. In order to obtain the posterior densities, non-informative and independent priors were employed. Performances of parameter estimations were verified by conducting several goodness-of-fit tests. The results showed that Bayesian MCMC method was slightly better than ML method in estimating GEV parameters.

Keywords: Annual maximum series; Bayesian MCMC; extreme rainfall analysis; extreme value distribution; generalized maximum likelihood

ABSTRAK

Pemodelan statistik bagi hujan melampau amat penting, memandangkan hasil dapatannya mampu membantu jurutera awam dan pakar runding untuk menjangka kebolehan struktur sesebuah bangunan untuk bertahan dalam situasi yang paling melampau. Data daripada siri maksimum tahunan (AMS) disuaikan menggunakan taburan nilai melampau teritlak (GEV) dengan menggunakan kaedah kebolehjadian maksimum (ML) dan kaedah simulasi Markov Chain Monte Carlo (MCMC) Bayes. Kelemahan kaedah ML dalam pengendalian sampel kecil diharap dapat diatasi dengan kaedah simulasi MCMC Bayes. Bagi mendapatkan taburan posterior, taburan prior tak-bermaklumat dan tak-bersandar digunakan. Padanan bagi parameter yang dicadangkan disahkan dengan menjalankan beberapa ujian kebagusan penyuaian (GOF). Hasilnya, didapati kaedah MCMC Bayes memberikan anggaran yang sedikit lebih baik berbanding kaedah ML bagi menganggar nilai-nilai parameter taburan GEV.

Kata kunci: Kaedah kebolehjadian maksimum; kajian hujan melampau; MCMC Bayes; siri maksimum tahunan; taburan nilai melampau teritlak

INTRODUCTION

Extreme rainfall event is often associated with climate change, which may be followed by a series of natural disasters such as flash floods and landslides. According to United Nations Framework on Climate Change (UNFCCC), climate change in Asia will affect water resources, agriculture and food security, ecosystems and biodiversity, human health and coastal zones. Malaysia is located in tropical climate zone; hence extreme rainfall is expected to take place regularly every year resulting from local tropical wet season. According to Department of Irrigation and Drainage (DID) Malaysia, it is estimated that about 29,720 km^2 or 9% of the total land area of Malaysia is prone to flood, affecting some 4.9 million people or 21% of the population. For a developing country like Malaysia, natural disasters will definitely cripple the country's productivity. Alam et al. (2011) found that changes in climatic factors have negative impacts on productivity of paddy cultivation in Malaysia.

It is undeniable that any extreme environmental event is unpredictable. Nevertheless, the impact of extreme rainfall events may be reduced by preventive measures based on the results from statistical analysis of extreme rainfall data, as suggested by Zin et al. (2009a). The main objective of extreme rainfall modeling was to estimate the values of return levels that might occur for the next 10, 50 or maybe 100 years, based on 10-year or 30-year history.

The two approaches widely used in data selection for extreme rainfall are annual maximum series (AMS) approach and peak over thresholds series (POT) approach (also known as partial duration series, PDS). The AMS method involves selection of the highest value of rainfall observed each year, whereas POT approach involves choosing observations that exceed a certain level of predetermine threshold value (Zin et al. 2009a). POT method was developed in order to solve the data-wastage problems in AMS approach. However, according to Madsen et al. (1997) AMS is preferable since the procedure in selecting the suitable threshold in POT method sometimes can be complicated.

Extreme rainfall data need to be modeled by suitable statistical distributions that give the best inferences of the behavior of extreme rainfall. Examples of distributions often is able to used in extreme rainfall analysis are Gumbel, GEV, generalized Pareto distribution (GPD), generalized logistic distribution (GLO) and lognormal distribution. Several studies conducted on selecting the best-fit distribution for extreme rainfall data in Malaysia are by: Zalina et al. (2002) which suggested that GEV is the most suitable distribution for annual maximum rainfall in Peninsular Malaysia, Zin et al. (2009a) which concluded that annual extreme and partial duration series in Peninsular Malaysia are well fitted by GEV and GPD model respectively, Zin and Jemain (2009b) which found that majority of 50 rain gauge stations in Peninsular Malaysia followed GLO and Shabri et al. (2011) which identified that GLO and GEV are two most suitable distributions for representing statistical properties of extreme rainfall in Selangor.

There are several methods that can be used in parameter estimation for extreme value models such as graphical-based, moment-based, order statistics, likelihood-based and simulation-based. Moment-based methods were the most preferable methods used in previous studies on extreme rainfall in Malaysia such as L-moment method (Shabri et al. 2011; Zalina et al. 2002; Zin et al. 2009a; Zin & Jemain 2009b), LQ-moment method (Zin & Jemain 2009b) and TL-moment method (Shabri et al. 2011). The reason was mainly due to the problem of scarcity data; common when dealing with extreme value analysis for this reason. Hosking (1990) and Hosking et al. (1985) affirmed that parameter estimation via moment-based techniques may produce a better estimation than ML. In contrast, Coles (2001) considered ML as the best method because of its all-round utility and adaptability to model-change. This means that, the underlying methodology is essentially unchanged even though the estimating equation is modified. However, a study by Smith (1985) showed that asymptotic properties associated with ML estimator are violated when estimating GEV parameters due to the assumption of a restricted parameter space. In addition, Rao and Hamed (2000) mentioned that ML method was incapable to obtain estimates with small sample. As an alternative, Bayesian approach can be used in estimating the parameters of GEV. Despite this approach is increasingly popular in many areas of application, a challenge when adopting this approach is the computational difficulties. This may be solved by the application of Markov chain Monte Carlo (MCMC) simulations (Coles 2001).

In order to produce good estimates, a long-record of rainfall data is required. It is generally held that a quantile of return period T can be reliably estimated from a data record of length n only if $T \leq n$ (Hosking & Wallis 1997). Although extreme data are limited in nature as mentioned earlier, Bayesian inferences have the ability to incorporate other source of information via prior distribution. It is also considered as a complete inference (Coles 2001) since the accuracy of an inference can be derived from the posterior distribution. Moreover, Bayesian analysis is not dependent on regularity assumptions required by asymptotic theory of ML. Previous studies on Bayesian extreme rainfall analysis using MCMC were conducted by Coles (2001), Coles and Tawn (1996), Coles et al. (2003), Fawcett and Walshaw (2008) and Smith (2005).

The objective of this study was to evaluate the performances of Bayesian MCMC and ML methods in estimating parameters of GEV parameters. We focused on GEV distribution only in representing the distribution of annual maximum daily rainfall in Alor Setar. Performance for both methods were then verified by conducting several goodness-of-fit tests.

This paper is organized as follows. We begin with description of the data used in this study. Next, the probability distribution, statistical methods and goodness-of-fit tests involved in this study will be explained. Later on, the results of our analysis will be presented. Conclusion and a short discussion regarding plan for future works will be discussed in the final section.

Data

In the preliminary phase in applying Bayesian approach in modeling extreme rainfall data in Malaysia, daily rainfall data from Alor Setar rain gauge station is selected. Alor Setar is the state capital of Kedah, located at the north region of Peninsular Malaysia. DID Malaysia reported that a total area of 209 km^2 of Kedah is flood prone area, which affected about 124,000 people. Known as the 'rice bowl' of Malaysia, Kedah has a long wet season resulting from the tropical monsoon climate. Zin et al. (2010) found that the annual pattern of extreme rainfall in the peninsula is highly influenced by the northeast monsoon. In this study, daily rainfall data of AMS from year 1970 to 2008 was obtained from the DID Malaysia. Figure 1 displays the scatter plot / time series plot of annual maximum amount of daily rainfall in Alor Setar recorded for the 38 years were considered.

Method

Probability Distribution

Some of the earliest applications of statistical theory of extreme were to hydrology and to closely related problems in climatology (Katz et al. 2002). According to Zin et al. (2009a), Gumbel distribution is commonly used among hydrologist in describing the AMS due to its simplicity form. Katz et al. (2002) stated that AMS can be modeled by

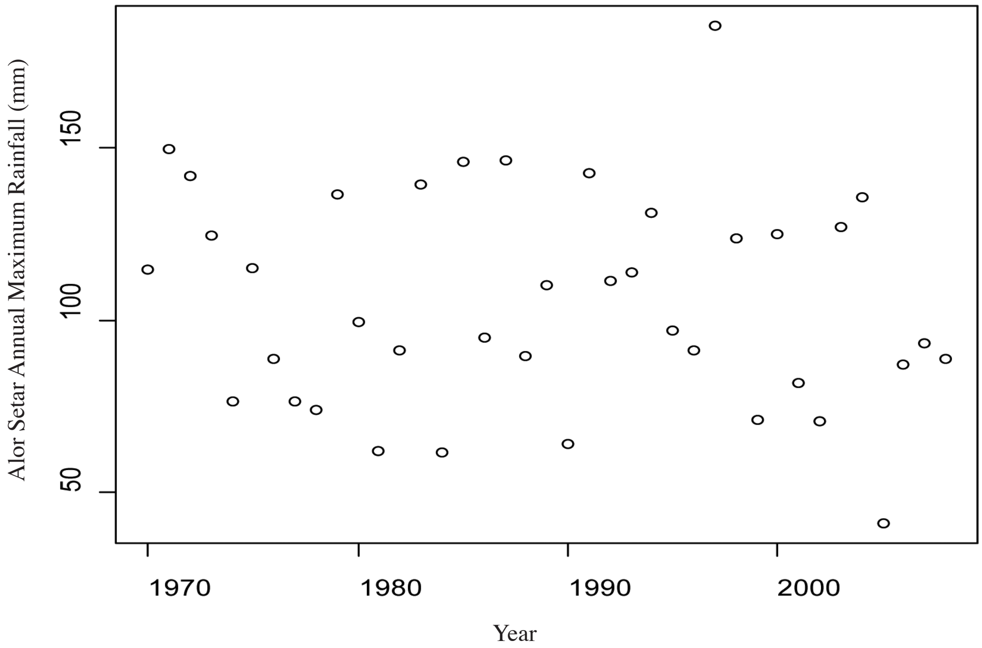

FIGURE 1. Annual maximum rainfall at Alor Setar

Gumbel or GEV distribution, whereas POT is suitable to be modeled by GPD distribution. Nevertheless, Koutsoyiannis (2003), Koutsoyiannis and Baloutsos (2000) revealed that Gumbel distribution may under estimate the largest extreme rainfall amount. Therefore, GEV distribution will be considered in this study.

Let $Z_1, \ldots, Z_n$ denote independent annual maximum rainfall observations having GEV probability density function:

$$f(z|\mu,\sigma,\xi) = \frac{1}{\sigma}\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]^{-1-\frac{1}{\xi}} \exp\left\{-\left[1+\xi\left(\frac{z_i-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}, \quad (1)$$

where μ is the location, σ is the scale and ξ is the shape parameters with parameter space $-\infty < \mu < \infty$, $\sigma > 0$ and $-\infty < \xi < \infty$ respectively. The estimate of extreme quantile, of annual maximum is:

$$q_r = \mu - \frac{\sigma}{\xi}\left\{1-\left[-\log(1-p)\right]^{-\xi}\right\}, \quad (2)$$

where q_r is the return level associated with the $1/r$-year return period. In order to obtain the parameter values, ML and Bayesian MCMC simulations methods will be used.

METHOD

In this paper, the discussion will be focused more on the Bayesian MCMC method instead of ML method. For reference on ML method, Rao and Hamed (2000) is a great source for describing ML method in estimating GEV parameters. As mentioned earlier, adopting Bayesian approach can be quite tedious. Coles (2001) argued that the integration of conditional probabilities used in obtaining the posterior distribution for a complex model can be problematic, even with the aid of sophisticated numerical integration techniques.

For the purpose of facilitating Bayesian calculation, MCMC aids in estimating parameters. Nevertheless, when applying MCMC a proposal distribution to generate simulated values need to be introduced. According to Nzoufras (2009), the choice of a proposal distribution is important as a poor choice will considerably delay convergence towards the equilibrium distribution (Roberts & Rosenthal 2001). The suitable acceptance rates in achieving high efficiency of MCMC simulation must be around 10 to 40%. Further information on MCMC simulation, may be found from Ntzoufras (2009) for application using WinBUGS, Gamerman and Lopes (2006) for a comprehensive introduction of MCMC simulation supported with R and WinBUGS computations and Gilks et al. (1996) for some theoretical background of MCMC and its implementations in medical statistics.

In this study, a combination of Gibbs sampling and Metropolis-Hasting scheme with random walk process is used to generate the distribution of interest. Therefore, the likelihood functions for $Z_1,\ldots,Z_n$ is given by:

$$L(\mu,\sigma,\xi{:}Z_1,\ldots,Z_m) = \prod_{i=1}^{n} f(z_i|\mu,\sigma,\xi), \quad (3)$$

Thus, the density of posterior distribution is:

$$f(\mu,\sigma,\xi|Z_1,\ldots,Z_m) \propto L(\mu,\sigma,\xi{:}Z_1,\ldots,Z_m) \times g(\mu,\sigma,\xi), \quad (4)$$

which can be represented as posterior ∝ prior × likelihood, where g is the prior distribution for GEV parameters. The prior distribution is used to represent a set of belief about the parameter of interest. In this study, the non-informative prior (also known as diffuse, flat or vague priors) distributions are used to indicate that the significant information related to the extreme rainfall in Alor Setar is still unavailable at the moment. This approach is based on Coles et al. (2003) and Fawcett and Walshaw (2008) which used Port Pirie annual maximum sea-levels data and Smith (2005) on annual maximum rainfall data for south-west England. The location, scale and shape parameters are assumed to be normally distributed with all means are equal to zero and variances equal to 1000, 100 and 10, respectively. Cases where informative priors were used in extreme rainfall data analysis for south-west England can be seen in Coles and Tawn (1996) and Smith (2005) with some modifications.

In general, the aim of extreme rainfall analysis is to estimate the expected values of the extreme rainfall in the future i.e. the future return level. According to Coles (2003), Bayesian analysis is preferable due to the prediction of return level which is based on predictive distribution can be estimated easily.

Let y denotes the future observation with probability density function:

$$h(y|Z_1,\ldots,Z_m) = \iiint f(y|\mu,\sigma,\xi)\, f(\mu,\sigma,\xi|Z_1,\ldots,Z_m)\,d\mu d\sigma d\xi. \tag{5}$$

Based on Coles (2001), this can be obtained via MCMC given that the posterior density (Equation 4) has been estimated by simulation. It other words, the simulated values for the three parameters of GEV obtained from MCMC simulations will be used to generate the posterior predictive distribution. After the removal of 'settling-in' or 'burn-in' period of the MCMC simulations, the procedure will produce a sample $\theta 1,\ldots,\theta B$ and the estimate of m-year return level is:

$$\Pr\{Y \le q_m | z_1, \ldots, z_n\} \approx \frac{1}{B}\sum_{i=1}^{B} \Pr\{Y \le q_m | \theta_i\}. \tag{6}$$

For more explanation on Bayesian predictive in extreme rainfall analysis, refer Coles (2001); Coles et al. (2003), Fawcett and Walshaw (2008) and Smith (2005). In this study, return values for 10, 25, 50 and 100-year are estimated using ML and Bayesian MCMC simulations.

GOODNESS-OF-FIT TEST

The performance between ML and Bayesian method in estimating GEV parameters and return levels of extreme rainfall in Alor Setar will be compared in this study. The selected GOF tests are relative root mean square error (RRMSE), relative absolute square error (RASE) and probability plot correlation coefficient (PPCC). The first two methods involve the assessment on the discrepancy between observed and estimated values under the assumed distribution while the third method involves measuring the correlation between the ordered values and the associated expected values (Zin et al. 2009a). The formulas are given as:

$$\text{RRMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_{i:n} - \hat{Q}(F_i)}{x_{i:n}}\right)^2} \tag{7}$$

$$\text{RASE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{x_{i:n} - \hat{Q}(F_i)}{x_{i:n}}\right| \tag{8}$$

$$\text{PPCC} = \frac{\sum_{i=1}^{n}(x_{i:n} - \bar{x})\left(\hat{Q}(F_i) - \bar{Q}(F_i)\right)}{\sqrt{\sum_{i=1}^{n}(x_{i:n} - \bar{x})^2 \sum_{i=1}^{n}\left(\hat{Q}(F_i) - \bar{Q}(F_i)\right)^2}} \tag{9}$$

where $x_{i:n}$ is the observed values for the ith order statistics of a random sample of size n, $\bar{Q}(F_i) = \frac{1}{n}\sum_{i=1}^{n}\hat{Q}(F_i)$ is the estimated quantile values associated with the ith Gringorton plotting position, F_1. The smallest values of RRMSE and RASE will indicate the best method. In contrast, the value of PPCC that is closest to 1 will be considered as the best method for explaining the behaviour of extreme rainfall in Alor Setar.

RESULTS

All simulated values for the three GEV parameters are found to converge within a parallel zone as shown in Figure 2, suggesting that no obvious tendencies or periodicities. As stated in the previous section, the variances for non-informative priors are chosen to be large enough in order to create flat priors. The density plots are displayed in Figure 3 where it can be seen that the shapes of the three estimated posterior densities are almost symmetrical. The wide-spread distributions as shown in this figure resulted from the large variances defined in the non-informative prior distributions.

From Table 1, it can be seen that the differences between GEV parameter estimation based on ML method and Bayesian MCMC simulation are very small. This is most probably due to the application of non-informative priors in the simulation. In particular, the estimated values for scale and shape parameters under ML are smaller than Bayesian. ML estimates could probably be a better method because the standard deviations of ML estimates for all parameters in this study are smaller than Bayesian.

Based on RRMSE and RASE (Table 2), it can be concluded that Bayesian method is slightly better than ML since the small RRMSE and RASE values obtained indicate small difference between the observed and estimated

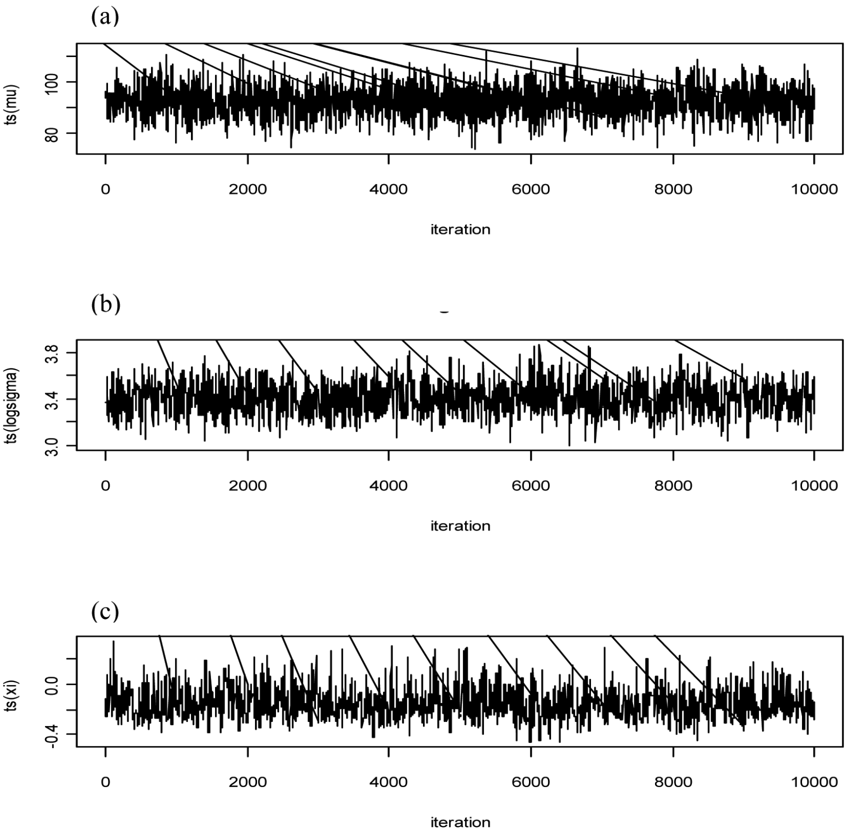

FIGURE 2. Trace plots for (a) location, (b) log scale and (c) shape parameters of GEV distribution for 10000 iterations

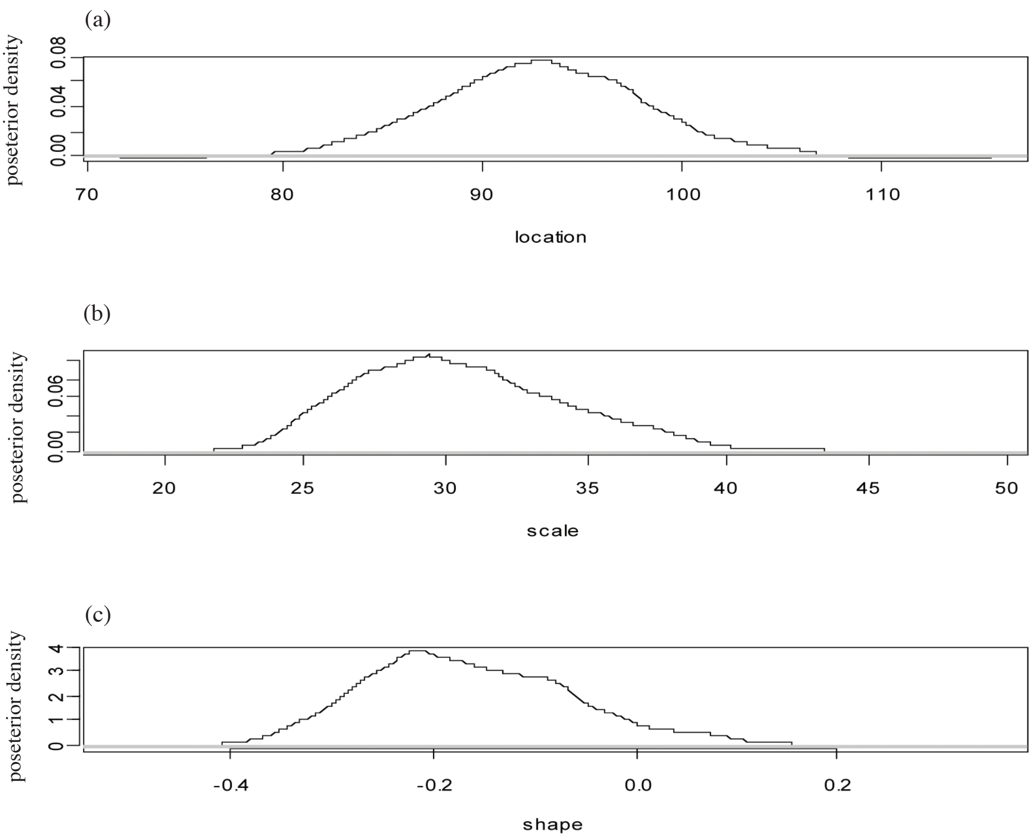

FIGURE 3. Posterior densities plots for (a) location, (b) scale and (c) shape of GEV parameters

TABLE 1. Estimates of GEV parameters

Method of estimation		Parameter		
		Location, μ	Scale, σ	Shape, ξ
ML	mean	93.6104319	29.3171622	-.2058092
	(std. dev.)	(5.2178781)	(3.7090043)	(0.1069889)
Bayesian	mean	92.7982	30.58968	-.1610848
	(std. dev.)	(5.379553)	(3.946326)	(0.1127518)

TABLE 2. Comparison of performance between Maximum Likelihood and Bayesian methods

Method of estimation	Goodness-of-fit test		
	RRMSE	RASE	PPCC
ML	0.1648584	1.306513	0.9300822
Bayesian	0.1461849	1.158966	0.9423074

values. Supported with PPCC test, the Bayesian method can be considered as the better method in estimating GEV parameters for Alor Setar.

All the posterior densities of the return levels given in Figure 4 are positively skewed. For this reason, it is advisable to use the posterior medians rather than posterior means. The return levels for four return periods are given in Table 3. All the return levels estimates of ML are smaller than the Bayesian estimates.

DISCUSSION AND CONCLUSION

The main objective of conducting this preliminary study was to verify whether the performance of parameter estimation can be improved when adopting Bayesian approach. ML and Bayesian MCMC are eventually closely related since the starting point as both method involved with likelihood function. As mentioned by Coles and Dixon (1999), the likelihood functions can be constructed

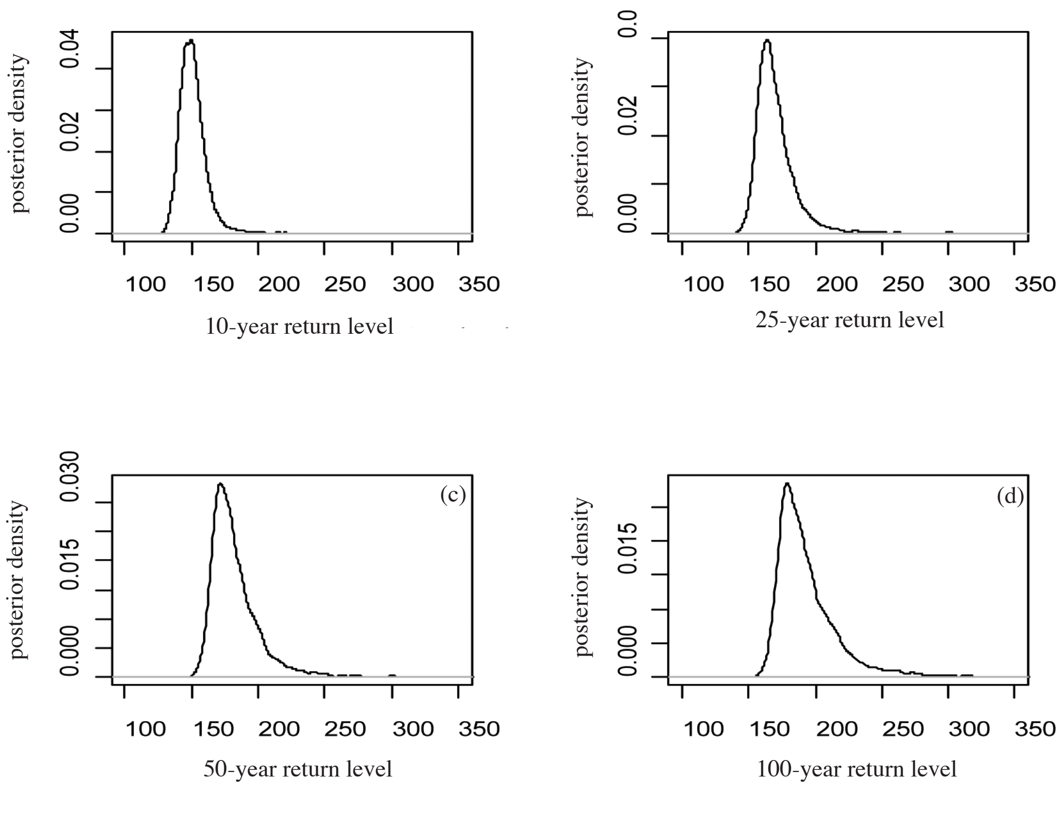

FIGURE 4. Posterior densities plots for (a) 10-year, (b) 25-year, (c) 50-year and (d) 100-year return levels (mm)

TABLE 3. Return levels (mm) for 10, 25, 50 and 100 year

Method of estimation	Return period (in years)			
	10	25	50	100
ML	146.4159	162.3079	172.2478	180.7889
Bayesian	149.6267	166.9308	178.2089	187.908

for complex modeling situations which can handle issues such as non-stationarity, covariates effects and regression modeling. Even though ML method has many advantages, its poor performance when dealing with small samples can be improved by conducting Bayesian MCMC.

In conclusion, Bayesian MCMC was a better method in describing the annual maximum rainfall of Alor Setar. The parameter estimation of GEV distribution was slightly better when using Bayesian compared to ML method. The small differences were due to the non-informative prior used in this study. Judiciously, Bayesian analysis did not give a radically different interpretation of the data, but provides a more convenient and direct way of managing and expressing uncertainties (Coles et al. 2003). It also has the ability to comprise other source of information in order to reduce the amount of uncertainties in the model. Furthermore, the prediction of future return levels of extreme rainfall can be derived easily by using the Bayesian predictive distribution.

Some issues that would be considered for future works in this study are the reliability of the model with the inclusion of expert knowledge for the usage of informative priors, several diagnostics tests in order to get high-efficiency of MCMC simulations and a Bayesian spatial extreme rainfall analysis constructed for Kedah or perhaps the northern region of Peninsular Malaysia.

ACKNOWLEDGEMENT

The authors are indebted to the staff of the Drainage and Irrigation Department Malaysia for providing the hourly rainfall data for this study. This research was funded by Universiti Kebangsaan Malaysia (UKM-GGPM-PI-028-2011).

REFERENCES

Alam, M.M., Siwar, C., Toriman, M.E., Molla, R.I. & Talib, B. 2011. Climate change induced adaptation by paddy farmers in Malaysia. *Mitig. Adapt. Strateg. Glob. Change* DOI: 10.1007/s11027-011-9319-5.

Coles, S. 2001. *An Introduction to Statistical Modeling of Extreme Values*. London: Springer.

Coles, S. & Dixon, M.J. 1999. Likelihood-based inference for extreme value models. *Extremes* 2: 5-23.

Coles, S. & Tawn, J.A. 1996. A Bayesian analysis of extreme rainfall data. *J. Appl. Statist.* 45: 463-478.

Coles, S., Pericchi, L. & Sisson, S. 2003. A fully probabilistic approach to extreme rainfall modeling. *J. Hydrology* 273: 35-50.

Fawcett, L. & Walshaw, D. 2008. Modeling environmental extremes. *Short course for the 19th annual conference of the International Environmentrics Society*, June 8, Kelowna, Canada.

Gamerman, D. & Lopes, H.F. 2006. *Markov Chain Monte Carlo: Stochastic simulation for Bayesian Inference*. Florida: Boca Raton.

Gilks, W.R., Richardson, S. & Spiegelhalter, D.J. 1996. *Markov Chain Monte Carlo in Practice*. London: Chapman and Hall.

Hosking, J.R.M. 1990. L-moments: Analysis and estimation of distribution using linear combinations of order statistics. *Journal of the Royal Statistical Society (B)* 52: 105-124.

Hosking, J.R.M. & Wallis, J.R. 1997 *Regional Frequency Analysis*: *An Approach Based On L-moments* London: Cambridge University Press.

Hosking, J.R.M., Wallis, J.R. & Wood, E.F. 1985. Estimation of the generalized extreme-value distribution by method of probability-weighted moments. *Technometrics* 27: 251-261.

Madsen, H., Rasmussen, P. & Rosbjerg, D. 1997. Comparison of annual maximum series and partial duration series methods for modeling extreme hydrologic events 1: at-site modeling. *Water Resour. Res*. 33: 747-757.

Ntzoufras, I. 2009. *Bayesian Modeling using WinBUGS*. New Jersey: John Wiley & Sons.

Katz, R.W., Parlange, M.B. & Naveau, P. 2002. Statistics of extremes in hydrology. *Adv. Water Resour.* 25: 1287-1304.

Koutsoyiannis, D. & Baloutsos, G. 2000. Analysis of a long record of annual maximum rainfall in Athens, Greece and design rainfall inferences. *Natural Hazards* 22: 31-51.

Koutsoyiannis, D. 2003. On the appropriateness of the Gumbel distribution in modeling extreme rainfall. *Proceedings of the ESF LESC Exploratory Workshop,* October 24-25, Bolonga, Italy.

Rao, A.R. & Hamed, H.K. 2000. *Flood Frequency Analysis*. Florida: Boca Raton.

Roberts, G.O. & Rosenthal, J.S. 2001. Optimal scaling for various Metropolis-Hasting algorithms. *J. Appl. Statist. Sci.* 16: 351-367.

Shabri, A.B., Zalina, M.D. & Ariff, N.M. 2011. Regional analysis of annual maximum rainfall using TL-moments method. *Theor. Appl. Climatol*. 104: 561-570.

Smith, E. 2005. Bayesian Modeling of Extreme Rainfall Data. Ph.D. dissertation, University of Newcastle upon Tyne, Newcastle (Unpublished).

Smith, R.L. 1985. Maximum likelihood estimation in a class of non-regular cases. *Biometrika* 72: 67-92.

Zalina, M.D., Desa, M.N., Nguyen, V.T.V. & Hashim, M.K. 2002. Selecting a probability distribution for extreme rainfall series in Malaysia. *Water Science and Technology* 45: 63-68.

Zin, W.Z.W., Jamaludin, S., Deni, S.M. & Jemain, A.A. 2010. Recent changes in extreme rainfall events in Peninsular Malaysia: 1971-2005. *Theor. Appl. Climatol*. 99: 303-314.

Zin, W.Z.W., Jemain, A.A., Ibrahim, K., Jamaludin, S. & Deni, S.M. 2009a. A comparative study of extreme rainfall in Peninsular Malaysia: with reference to partial duration and annual extreme series. *Sains Malaysiana* 38: 751-760.

Zin, W.Z.W. & Jemain, A.A. 2009b. The best fitting distribution of annual maximum rainfall in Peninsular Malaysia based on methods of L-moment and LQ-moment. *Theor. Appl. Climatol*. 96: 337-344.

Annazirin Eli*
Department of Science in Engineering
Faculty of Engineering
International Islamic University Malaysia
50728 Gombak, Kuala Lumpur
Malaysia

Mardhiyyah Shaffie & Wan Zin Wan Zawiah
School of Mathematical Siences
Faculty of Science and Technology
Universiti Kebangsaan Malaysia
43600 Bangi, Selangor
Malaysia

*Corresponding author; email: annazirin@iium.edu.my

Received: 10 October 2011
Accepted: 18 June 2012